TRINITY COLLEGE

**Semester One Examination, 2017**

**Question/Answer Booklet**

If required by your examination administrator, please place your student identification label in this box

# MATHEMATICS APPLICATIONS

# UNITS 3 and 4

## Section Two: Calculator-assumed

Student Number: In figures | | | | | | | | |

In words _____

Your name _____

## Time allowed for this section

Reading time before commencing work: ten minutes
Working time for this section: one hundred minutes

## Materials required/recommended for this section

***To be provided by the supervisor***

This Question/Answer Booklet
Formula Sheet (retained from Section One)

***To be provided by the candidate***

Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: drawing instruments, templates, notes on two unfolded sheets of A4 paper, and up to three calculators

## Important note to candidates

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised notes or other items of a non-personal nature in the examination room. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

## Structure of this paper

| Section | Number of questions available | Number of questions to be answered | Working time (minutes) | Marks available | Percentage of exam |
|---|---|---|---|---|---|
| Section One: Calculator-free | 7 | 7 | 50 | 50 | 35 |
| Section Two: Calculator-assumed | 11 | 11 | 100 | 101 | 65 |
| | | | **Total** | 151 | 100 |

## Instructions to candidates

1. The rules for the conduct of Trinity College examinations are detailed in the *Instructions to Candidates* distributed to students prior to the examinations. Sitting this examination implies that you agree to abide by these rules.

2. Write your answers in this Question/Answer Booklet.

3. You must be careful to confine your response to the specific question asked and to follow any instructions that are specific to a particular question.

4. Spare pages are included at the end of this booklet. They can be used for planning your responses and/or as additional space if required to continue an answer.
   - Planning: If you use the spare pages for planning, indicate this clearly at the top of the page.
   - Continuing an answer: If you need to use the space to continue an answer, indicate in the original answer space where the answer is continued, i.e. give the page number. Fill in the number of the question that you are continuing to answer at the top of the page.

5. **Show all your working clearly**. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.

6. It is recommended that you **do not use pencil**, except in diagrams.

7. The Formula Sheet is **not** to be handed in with your Question/Answer Booklet.

**Section Two: Calculator-assumed** **65% (101 Marks)**

This section has **eleven (11)** questions. Answer **all** questions. Write your answers in the spaces provided.

Working time for this section is 100 minutes.

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**Question 8** **(6 marks)**

The weight, $W_n$ kg, of flour produced by a mill that needs to be sent to the packing department is given by $W_{n+1} = W_n + 1.25,\ W_0 = 7.5$, where $n$ is the number of minutes after 5 am.

(a) Complete the table below. (2 marks)

| $n$ | 0 | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|---|
| $W_n$ | 7.5 | | | | | |

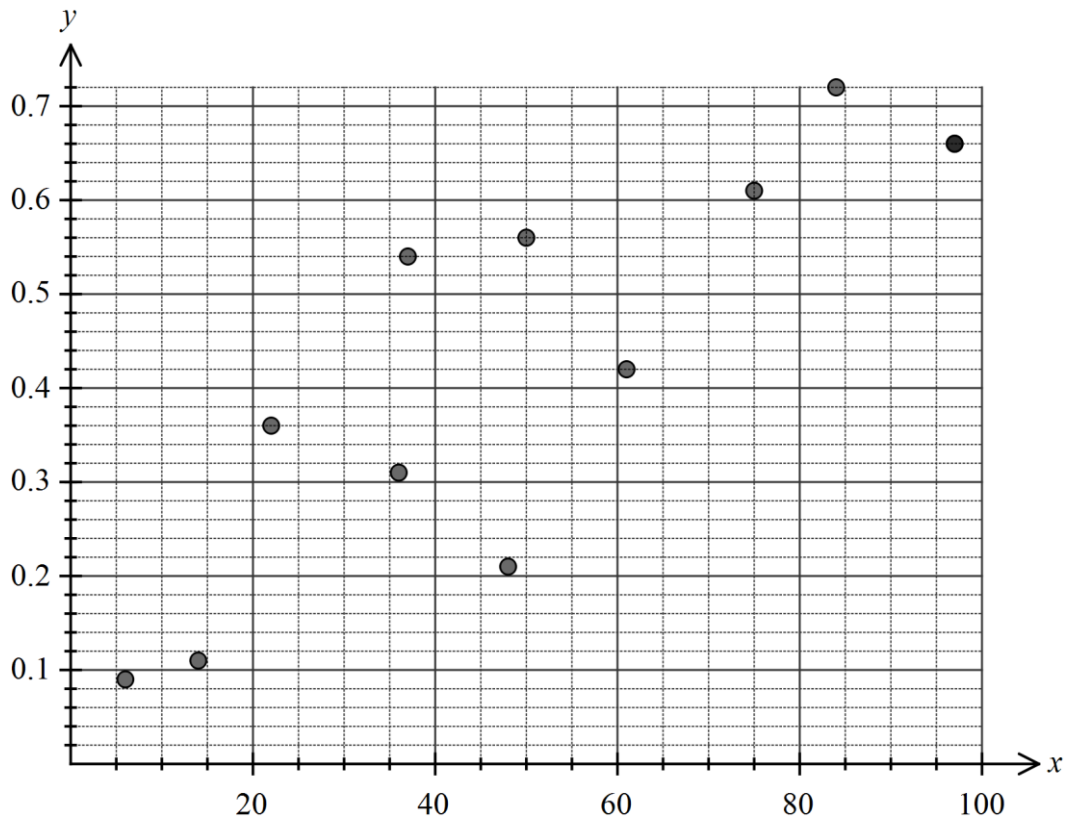
(b) Calculate the weight of flour at 6 am. (2 marks)

(c) At what time will the weight of flour reach 150 kg? (2 marks)

**Question 9** **(13 marks)**

Agricultural researchers collected data on the amount of rainfall ($x$ mm) and the yield of cucumbers ($y$ kg per square metre) over several seasons at a farm. Some of their data is shown in the table and scatterplot below.

| Rainfall, $x$ | 22 | 84 | 97 | 48 | 14 | 37 | 97 | 50 | 61 | 75 | 36 | 6 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Yield, $y$ | 0.36 | 0.72 | 0.66 | 0.21 | 0.11 | 0.54 | 0.66 | 0.56 | 0.42 | 0.61 | 0.31 | 0.09 |




(a) Calculate the correlation coefficient for the data, and comment on how its value is reflected in the scatterplot above. (3 marks)

(b) What percentage of the variation in the yield can be explained by the variation in the rainfall? (2 marks)

**See next page**

(c) Determine the equation for the least-squares line that models the data. State the slope and vertical intercept correct to four decimal places. (2 marks)

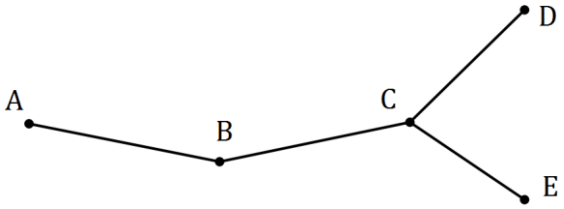
(d) Draw the least-squares line on the scatterplot by first calculating two points that lie on the line. Clearly indicate these points. (3 marks)

(e) Estimate the cucumber yield in a season that has 64 mm of rainfall and comment on the reliability of this value. (3 marks)

**See next page**

**Question 10** **(8 marks)**

A simple connected graph is drawn below.

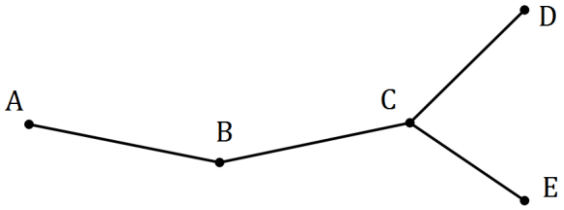



(a) List, in order, the vertices of a closed walk on the graph of length 8 that visits all vertices and ends at A.

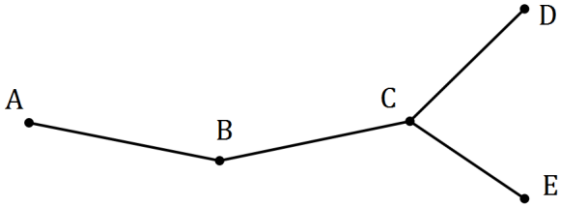
(2 marks)

(b) Explain why the walk in (a) is not a Hamiltonian cycle. (2 marks)

(c) Add one edge and one face to the copy of the graph below, so that the new graph contains a Hamiltonian path and mark this path on the graph. (2 marks)




(d) Add two edges and two faces to the copy of the graph below, so that the new graph does not contain a Hamiltonian path or cycle. (2 marks)

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**Question 11** **(9 marks)**

The value of a machine used in a factory is recorded at the start of each year.

| Year | 2014 | 2015 | 2016 |
|---|---|---|---|
| Value of machine ($) | 6 875 | 5 500 | 4 400 |

(a) Explain why the three values in the table form a geometric sequence. (2 marks)

(b) What is the annual percentage rate of depreciation of the machine? (1 mark)

Assume that the machine continues to depreciate at the same rate.

(c) Determine a rule for $V_n$, the value of the machine $n$ years after 2014. (2 marks)

(d) Determine the value of the machine at the start of the year 2020. (2 marks)

(e) The machine will be replaced when its value at the start of the year falls below $500. Determine which year this will be. (2 marks)

**See next page**

**Question 12** **(11 marks)**

In a recent study of artists who asked for a piece of their work to be included in an exhibition, each artist was classified by the variables (i) the state they worked in and (ii) whether their piece of work was accepted by the judges.

The table below shows the number of artists in each category.

| | State | NSW | VIC | QLD | WA | Total |
|---|---|---|---|---|---|---|
| Work accepted? | Yes | 8 | 27 | | 8 | 64 |
| | No | | 86 | 143 | 39 | |
| | Total | | | | | 440 |

(a) Complete the missing values and totals in the table above. (4 marks)

(b) To identify the presence of an association between these two variables, explain why the state the artist worked in should be used as the explanatory variable. (2 marks)

**See next page**

(c) Rounding percentages to the nearest whole number, complete the percentaged two-way table below so that it may be used to identify the presence of an association between the categorical variables. (3 marks)

| | State | NSW | VIC | QLD | WA |
|---|---|---|---|---|---|
| Work accepted? | Yes | | | | |
| | No | 93% | | | |

(d) Comment on the presence of an association between the two variables. (2 marks)

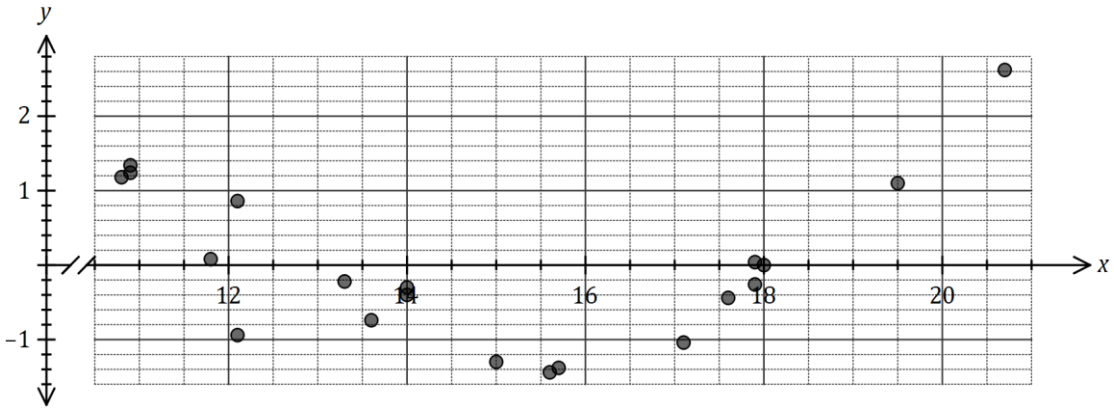
**See next page**

**Question 13** **(6 marks)**

A student was trying to decide whether fitting a linear model to their data was an appropriate choice. They calculated the least-squares line through the 30 points to be $\hat{y} = 2.4x - 12.5$ and the correlation coefficient $r = 0.98$.

(a) Explain why constructing a residual plot would help the student decide. (2 marks)

The residual plot for the student's data is shown below.




(b) One residual is missing from the plot, corresponding to the original data point $(20.5, 37.3)$.

Calculate the residual for this point and add it to the residual plot. (3 marks)

(c) What conclusion should the student draw about the appropriateness of the linear model? (1 mark)

**Question 14** **(10 marks)**

Following the analysis of data collected from a group of women aged between 22 and 38, a strong, linear relationship between their age ($x$, in years) and percentage chance of conception ($y$, percent) in any given month was observed. The coefficient of determination between the variables was 0.87 and the equation of the least-squares line was $\hat{y} = -0.88x + 45.8$.

(a) Determine the correlation coefficient between $x$ and $y$. (2 marks)

(b) Estimate the monthly percentage chance of conception of a woman aged

(i) 18 years. (1 mark)

(ii) 35 years. (1 mark)

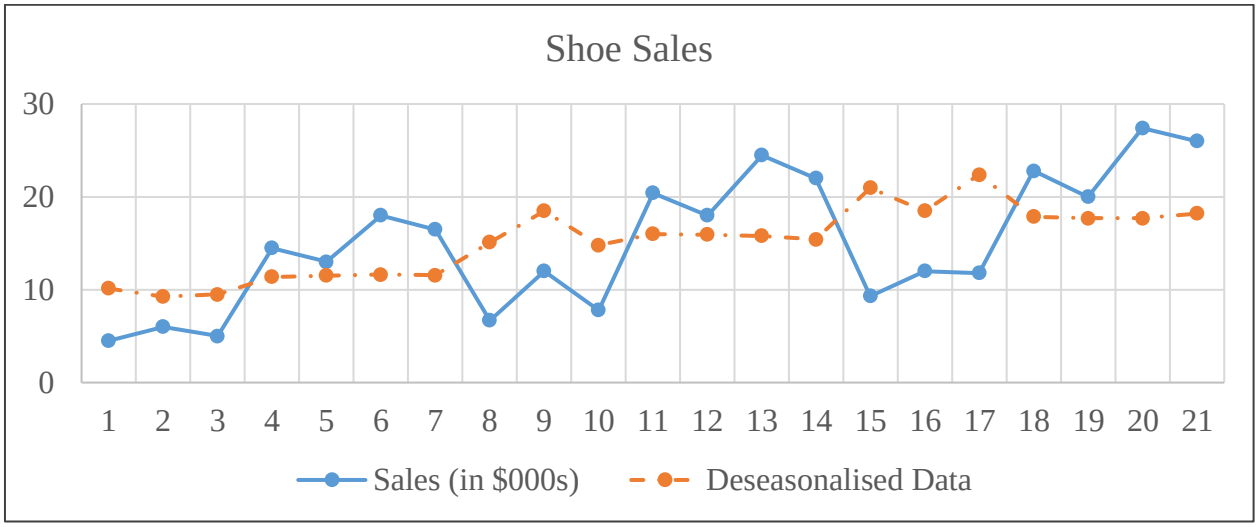
(c) Comment, with reasoning, on the reliability of each of your estimates in (b). (4 marks)

(d) Describe the meaning of the slope of the least-squares lines in the context of this question. (2 marks)

**See next page**

**Question 15** **(10 marks)**

The daily sales for a shoe store over three weeks are shown in the graph and tables below.




| Week | Day | Time ($t$) | Sales (in $000s) | Cycle Mean | Seasonal Effect | Deseasonalised Data |
|---|---|---|---|---|---|---|
| 1 | Monday | 1 | 4.5 | 11.07 | 40.65% | 10.1 |
| | Tuesday | 2 | 6.0 | | 54.19% | 9.3 |
| | Wednesday | 3 | 5.0 | | 45.16% | 9.5 |
| | Thursday | 4 | 14.5 | | **A** | **B** |
| | Friday | 5 | 13.0 | | 117.42% | 11.5 |
| | Saturday | 6 | 18.0 | | 162.58% | 11.6 |
| | Sunday | 7 | 16.5 | | 149.03% | 11.6 |
| 2 | Monday | 8 | 6.7 | 15.91 | 42.10% | 15.1 |
| | Tuesday | 9 | 12.0 | | 75.40% | 18.5 |
| | Wednesday | 10 | 7.8 | | 49.01% | 14.8 |
| | Thursday | 11 | 20.4 | | 128.19% | 16.0 |
| | Friday | 12 | 18.0 | | 113.11% | 15.9 |
| | Saturday | 13 | 24.5 | | 153.95% | 15.8 |
| | Sunday | 14 | 22.0 | | 138.24% | 15.4 |
| 3 | Monday | 15 | 9.3 | 18.47 | 50.35% | 21.0 |
| | Tuesday | 16 | 12.0 | | 64.97% | 18.5 |
| | Wednesday | 17 | 11.8 | | 63.88% | 22.4 |
| | Thursday | 18 | 22.8 | | 123.43% | 17.9 |
| | Friday | 19 | 20.0 | | 108.28% | 17.7 |
| | Saturday | 20 | 27.4 | | 148.34% | 17.7 |
| | Sunday | 21 | 26.0 | | 140.76% | 18.2 |

**Seasonal Indices**

| Monday | Tuesday | Wednesday | Thursday | Saturday | Sunday |
|---|---|---|---|---|---|
| 0.4436 | 0.6485 | 0.5269 | 1.2753 | 1.5496 | **C** |

**See next page**

(a) Describe any unseasonal fluctuations. (1 mark)

(b) Calculate the missing values A, B and C in the tables on page 12. (3 marks)

The least squares regression line, using the deseasonalised data is $\hat{y} = 0.5221t + 9.4890$

(c) Using the line of regression, predict the sales for Wednesday of week 4 and comment on your prediction. (3 marks)

(d) The store will receive a bonus if during week 4 they can exceed $37 000 in sales on any day. If sales continue in the current trend, is the store likely to receive this bonus? Justify. (3 marks)

**See next page**

**Question 16** **(12 marks)**

As part of a trial to reintroduce woylies (an endangered species of mammals) to a wildlife reserve, researchers modelled the expected size of a woylie population, $P_n$, using the rule below where $n$ is the number of months since the trial began.

$$P_{n+1} = 0.85P_n + 30, \qquad P_0 = 20.$$

(a) State

(i) the size of the woylie population at the start of the trial. (1 mark)

(ii) the number of woylies added to the reserve each month. (1 mark)

(iii) the percentage loss of existing woylies in the reserve each month. (1 mark)

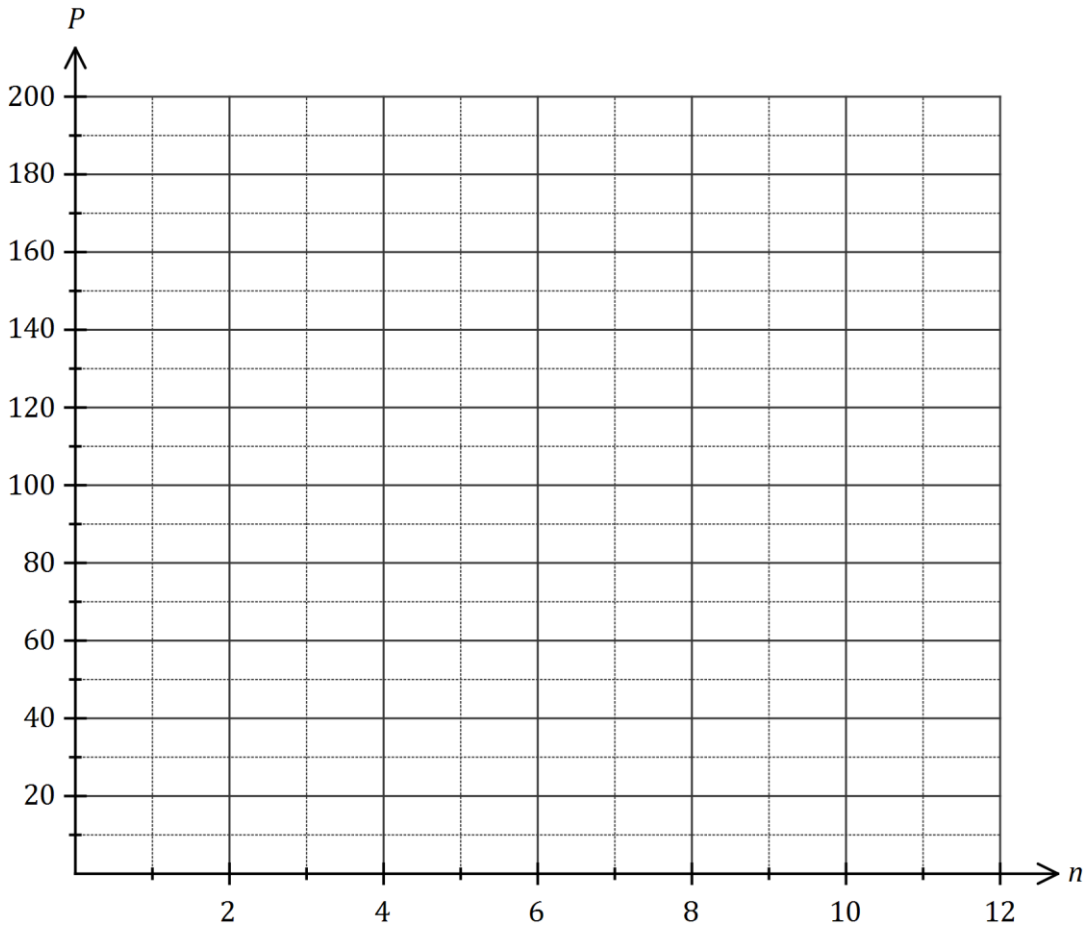
(b) Complete the missing values in the table below to show the expected number of woylies over the first six months. (2 marks)

| $n$ | 0 | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|---|
| $P_n$ | | | 70 | 89 | 106 | | |

(c) Determine the expected size of the woylie population after three years. (2 marks)

**See next page**

(d) Graph the population of woylies on the axes below for $0 \leq n \leq 12$. (3 marks)




(e) Use the model to describe how the size of the woylie population in the reserve will change over the first three years. (2 marks)

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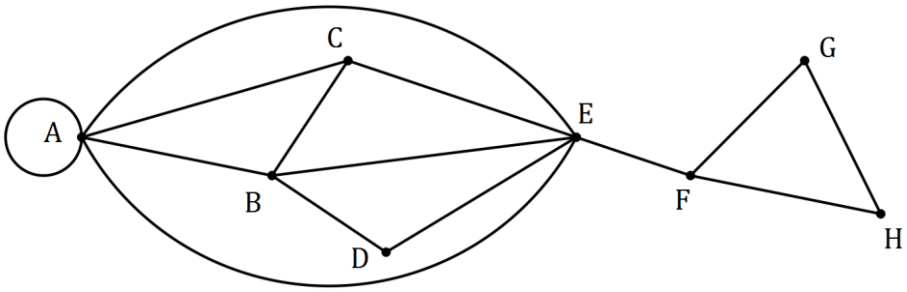
**Question 17** **(9 marks)**

The graph below represents 14 canals that meet at locations $A$ to $H$.




(a) A canal enthusiast noticed that the graph contained a semi-Eulerian trail. State the two properties of a walk that make a semi-Eulerian trail. (2 marks)

(b) What property of a connected graph indicates the existence of a Eulerian trail, a semi-Eulerian trail or neither? (2 marks)

(c) Suggest a suitable starting point for the canal enthusiast to begin a semi-Eulerian trail and indicate where they will finish the trail. (2 marks)

(d) Draw a subgraph of the above graph that is simple, connected, has no bridges and has 8 edges. (3 marks)

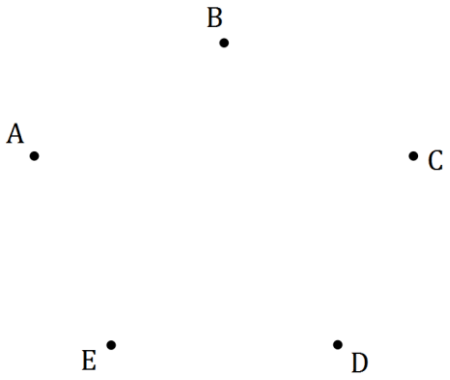
**See next page**

**Question 18** **(7 marks)**

Five children $(A, B, C, D, E)$ are playing a game of hide and seek. The directed edges, shown in the adjacency matrix $M$ below, represent whether the child in a row knows the location of the child in a column.

| $M$ | $A$ | $B$ | $C$ | $D$ | $E$ |
|---|---|---|---|---|---|
| $A$ | 0 | 1 | 1 | 0 | 0 |
| $B$ | 0 | 0 | 1 | 0 | 1 |
| $C$ | 0 | 0 | 0 | 1 | 1 |
| $D$ | 1 | 1 | 0 | 0 | 0 |
| $E$ | 0 | 0 | 0 | 1 | 0 |

(a) Construct a digraph to show the above information. (3 marks)




(b) Given the two matrices:

$$M^2 = \begin{bmatrix} 0 & 0 & 1 & 1 & 2 \\ 0 & 0 & 0 & 2 & 1 \\ 1 & 1 & 0 & 1 & 0 \\ 0 & 1 & 2 & 0 & 1 \\ 1 & 1 & 0 & 0 & 0 \end{bmatrix} \quad M^2 + M = \begin{bmatrix} 0 & 1 & 2 & 1 & 2 \\ 0 & 0 & 1 & 2 & 2 \\ 1 & 1 & 0 & 2 & 1 \\ 1 & 2 & 2 & 0 & 1 \\ 1 & 1 & 0 & 1 & 0 \end{bmatrix}$$

(i) If child $D$ wanted to locate child $E$ with the help of one other child, could they? Explain how an element of $M^2$ can help justify your answer. (2 marks)

(ii) Two children are unable to locate all the other children, even with the help of one other child. Who are these two, and who can't they locate? (2 marks)

**END OF QUESTIONS**

**Additional working space**

Question number: _____

**Additional working space**

Question number: _____